A Lower Limit on the Interaction Between Symmetrical Odd-Mode Fringing Capacitances

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Let us denote the familiar odd-mode fringing capacitance which is plotted in graphs given by Getsinger [1] by C_{f0}'. It is further defined by Fig. 1(b) in the limit as the magnetic wall tends to infinity on the right. We also denote the corresponding fringing capacitance of the symmetrical coaxial structure of Fig. 1(a) by C_{f0}. When the dimensions of Fig. 1(a) and (b) are such that $s' = s$, it is convenient to denote the difference between C_{f0}' and C_{f0} by ΔC_{f0}, and define it as the interaction between the symmetrical odd-mode fringing capacitances. Clearly, ΔC_{f0} has the property of approaching zero as $w/(b-t) \to 0$ [w is defined in Fig. 1(a)]. Moreover, whenever ΔC_{f0} is known, C_{f0} is also known.

In general, ΔC_{f0} is a function of w, t, s, and b, and it is convenient to think of it as a function of $w/(b-t)$ depending on two independent parameters, s/b and t/b. In a recent note [2], the writer identified the upper limit of this family of curves. Denoting it by $\bar{\Delta}C_{f0}$, it was shown that

$$\bar{\Delta}C_{f0} = \frac{K'}{K} + \frac{2}{\pi}\log\frac{kK}{\pi 2} \tag{1}$$

where k, K, and K' are defined by

$$\frac{K'}{K} = \frac{w}{b-t}. \tag{2}$$

This limit occurs when $s/b \to 0$ and is independent of t/b. This letter identifies the lower limit of the family denoted by $\boldsymbol{\Delta}C_{f0}$. It is given by

$$\boldsymbol{\Delta}C_{f0} = \frac{K'-E'}{E} + \frac{2}{\pi}\left(1 - \log\frac{8E}{K}\right) \tag{3}$$

where K, E, and E' are defined by

$$\frac{K'-E'}{E} = \frac{w}{b-t}. \tag{4}$$

This limit occurs when $t/b \to 1$ and is independent of s/b. Both $\bar{\Delta}C_{f0}$ and $\boldsymbol{\Delta}C_{f0}$ are plotted as a function of $w/(b-t)$ in Fig. 2. It is not difficult to show from (1) and (3) that $\bar{\Delta}C_{f0}$ and $\boldsymbol{\Delta}C_{f0}$ ultimately decrease as $\exp[-2\pi w/(b-t)]$, and that they then differ by a factor of $\exp(-4)$. They are substantially parallel as soon as $w/(b-t) > 0.1$.

The derivation of (3) is somewhat more involved than that of (1) although it uses the same type of limiting procedures. In the interest of brevity and clarity, this letter will only outline the steps in the derivation.

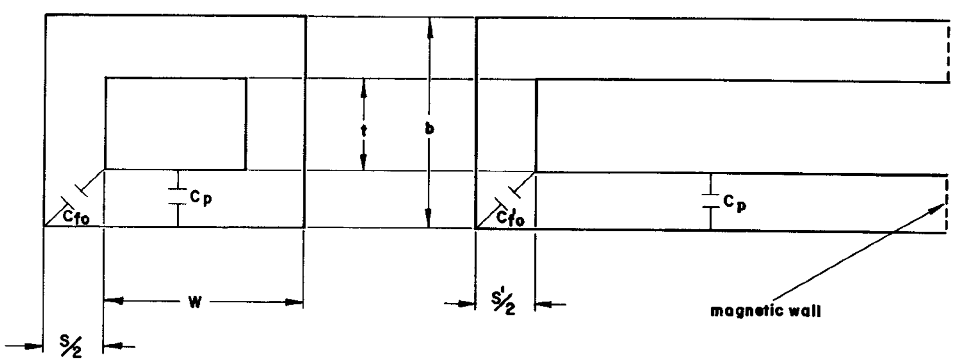
The arguments require two results which may be shown on the basis of physical intuition and are amply supported by examples:

1) ΔC_{f0}, for fixed t/b and $w/(b-t)$, is a decreasing function of s/b;
2) ΔC_{f0}, for fixed s/b and $w/(b-t)$, is a decreasing function of t/b.

In the derivation of (1) in [2], it was possible to find the limiting values of ΔC_{f0} as $s/b \to 0$ for any value of $t/b > 0$. It was then somewhat of a surprise that the limit turned out to be independent of t/b. The derivation of (3) is more involved because there is no known exact way of finding the limit of ΔC_{f0} as $t/b \to 1$ for arbitrary values of s/b. There are values of s/b, however, for which this limit can be evaluated. When $s/b = \infty$, Bates [3] has shown how the capacitance of the system can be found for arbitrary t/b and $w/(b-t)$. When ΔC_{f0} is found from these formulas in the limit as $t/b \to 1$, (3) results. Also using methods introduced in [2], the limiting value of ΔC_{f0} can also be found as $t/b \to 1$ when $s/b \to 0$. This turns out to be (3) again, and one concludes from 1) that the limit of ΔC_{f0} as $t/b \to 1$ is independent of s/b. From 2) it follows that this limit, called $\boldsymbol{\Delta}C_{f0}$, is the lower limit of all curves of the two-parameter family.

Curves of ΔC_{f0} for which $t/b = 0$ can also be calculated exactly [4], and three curves for which $s/b = 0.1$, 0.3, and 0.5 are also plotted in Fig. 2 as dashed lines. It has previously been seen, of course, that curves of ΔC_{f0} for $s/b = \infty$ can be calculated exactly, and five curves for which $t/b = 0$, 0.1, 0.2, 0.4, and 0.6 are plotted as solid lines in this figure. These new curves constitute secondary limiting values for ΔC_{f0}. For example, if we required ΔC_{f0} for $t/b = 0.4$ and $s/b = 0.5$, we would know that it lies below the dotted curve marked (0,0.5) and above the solid curve marked (0.4,∞). These curves are substantially parallel for $w/(b-t) > 0.01$. Now ΔC_{f0} can be readily found exactly when $w = 0$ [4], and for most purposes the desired curve can be estimated with sufficient accuracy. There is another exact point on this curve which can be found by exact methods [5], [6], and with the two points the desired curve for ΔC_{f0} can then be plotted with considerable accuracy even when $w/(b-t) < 0.01$.

The present author is indebted to one of the reviewers of this letter for a reference to a paper by Pregla [7] which is concerned with the same problem. He has plotted what amounts to curves of C_{f0}' as a function of s/b for discreet values of t/b for the two known cases $w = 0$ and $w = \infty$. He has shown how one may interpolate between these curves to obtain C_{f0} for a given value of w/b using the one exact value given by [6]. The use of Fig. 2 for the same purpose seems to the writer to have the following advantages: 1) it does not require a double interpolation; 2) it has greater accu-



(a) (b)

Fig. 1. Definition of parameters.

Manuscript received January 17, 1974; revised May 17, 1974, and August 29, 1974.
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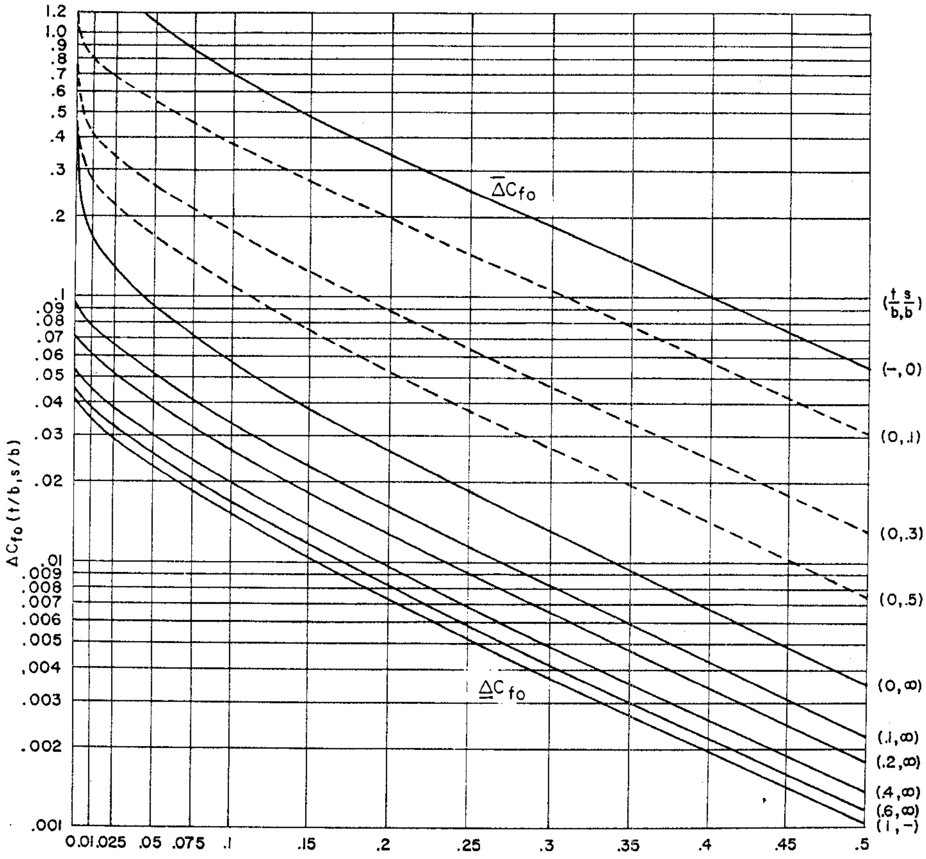

Fig. 2. A universal diagram for ΔC_{f0}.

racy because it plots directly the difference between nearly equal quantities; 3) it includes the limiting cases $t/b = 1$ and $s/b = 0$; 4) two points on each interpolated curve instead of one are known; and 5) the curves of the figure are substantially parallel and straight.

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Contributors

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